VIII KYIV INTERNATIONAL PHYSICS AND MATHEMATICS FESTIVAL

Oral mathematics Olympiad. 10th form. Main problems

1. Let $ABCD$ be a trapezium with $AD \parallel BC$. Suppose M and N are, respectively, points on the sides AB and CD such that $\angle BAN = \angle CDM$. Prove that $\angle BNA = \angle CMD$.

2. Find all functions $f:\mathbb{R} \to \mathbb{R}$ so that:

(1) $f(2x) = f(x+y)f(y-x) + f(x-y)f(-x-y)$ for all real x and y;

(2) $f(x) \geq 0$ for all real x.

3. Determine all primes p such that $5^p + 4p^4$ is the square of an integer.

4. Consider a 2009×2009 chessboard. Let n be the smallest number of rectangles that can be drawn on the chess board so that the sides of every cell of the board is contained in the sides of one of the rectangles. Find the value of n.

May 8th, 2009

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5. If the polynomial $f(x)=x^6+a_1x^5+a_2x^4+a_3x^3+a_4x^2+a_5x+3$ with real coefficients possesses 6 negative roots (some of them can be equal), show that $f(2)\geq 27^2$.

6. In the acute triangle ABC M is a point in the interior of the segment AC, and N is a point on the extension of the segment AC such that $MN=AC$. Let D and E be the feet of the perpendiculars from M and N onto the lines BC and AB respectively. Prove that the orthocenter of $\triangle ABC$ lies on the circumcircle of $\triangle BED$.

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